APPLICATION OF NONLINEAR HEREDITARY STRAIN THEORY TO DESCRIPTION OF STRESS RELAXATION IN METALS, AND CONVERSION OF RELAXATION DATA TO CREEP

G. S. Vorotnikov and L. Kh. Papernik

The possibility of describing relaxation and creep processes in metals and the possibility of direct conversion of data obtained in one type of tests to another, within the framework of the theory of hereditary creep based on the application of the Rabotnov nonlinear integral equation [1], are investigated.

The relationship between the stress $\sigma(t)$ and strain $\varepsilon(t)$ appears in the one-dimensional case as [1]

$$\varphi[\varepsilon(t)] = \sigma(t) + \lambda \int_0^t K(t-\tau)\sigma(\tau)\,d\tau \tag{1}$$

Here $\varphi(\varepsilon)$ is the strain function, which is nonlinear in the general case; $K(t-\tau) > 0$ is a monotonic decreasing influence function; λ is a numerical factor.

Equation (1) is a fairly general form of the relationship between strain and stress. In particular, when $\varphi(\varepsilon) = E\varepsilon$, Eq. (1) becomes the conventional linear hereditary law of strain

$$\varepsilon(t) = \frac{1}{E}\left[\sigma(t) + \lambda \int_0^t K(t-\tau)\sigma(\tau)\,d\tau\right] \tag{2}$$

In the case of creep, when $\sigma(t) = \sigma_0 = \text{const}$, Eq. (1) becomes

$$\varphi[\varepsilon(t)] = \sigma_0[1 + G(t)] \qquad \left(G(t) = \int_0^t K(\tau)\,d\tau\right) \tag{3}$$

Equation (3) is expressed in the form of a bundle of similar curves at fixed times $t_1, t_2, \ldots$ in σ, ε coordinates at different initial stress levels $\sigma_{01}, \sigma_{02}, \ldots$. Hence, the validity of Eq. (1) is clear when isochronous creep curves are similar.

The function $1 + \lambda G(t)$ yields similitude factors for fixed values of t. When $t = 0$, we have the instantaneous strain curve $\sigma = \varphi(\varepsilon)$.

The relaxation law will be expressed, when $\varepsilon = \varepsilon_0 = \text{const}$, by the equation

$$\sigma(t) = \sigma_0[1 - \lambda R(t)] \tag{4}$$

Here $\sigma_0 = \varphi(\varepsilon_0)$, and R(t) is the integral of the resolvent kernel. The power-law kernels At^{α} (where A is a numerical coefficient and $-1 < \alpha < 0$) are more generally used in application to metals. Since the parameter α is negative, Eq. (3) possesses an integrable singularity at zero. The existence of this singularity is due to the behavior of the material when loaded.

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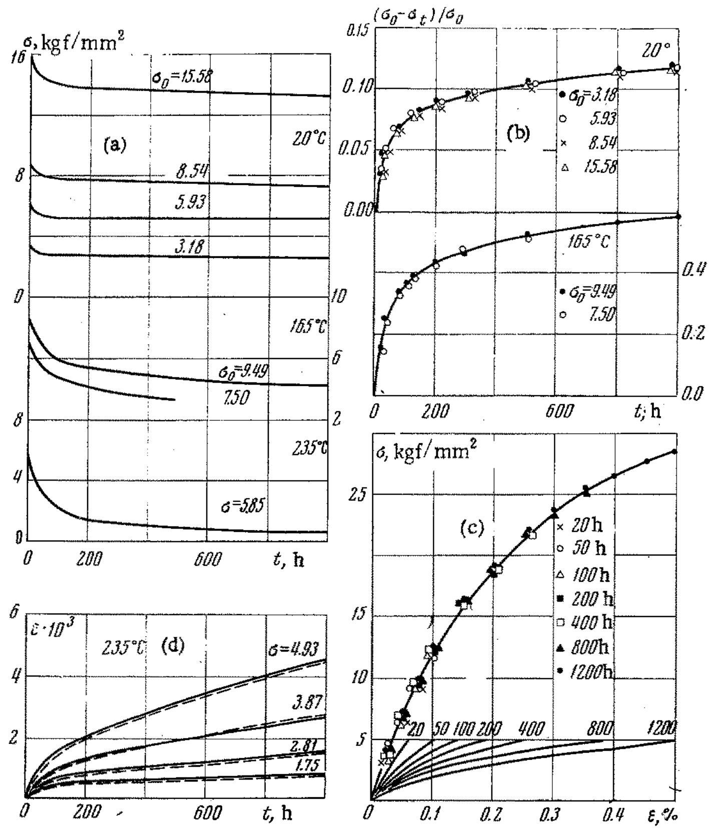

Fig. 1

We assume

$$K(t-\tau)=\frac{(t-\tau)^{\alpha}}{\Gamma(\alpha+1)}$$

Here $\Gamma(\alpha + 1)$ is the Eulerian gamma function. The resolvent will then be the same as Rabotnov's ϑ function [1]

$$\vartheta_{\alpha}(\beta,\, t-\tau)=(t-\tau)^{\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{n}\beta^{n}(t-\tau)^{(\alpha+1)n}}{\Gamma[(\alpha+1)(n+1)]} \tag{5}$$

Numerous properties of this function are detailed in [1, 2], and the feasibility of utilizing this function as the kernel of integral operators in problems in the linear theory of creep and relaxation is discussed.

Below we attempt to apply the mathematical tools provided by these functions to the study of relaxation and creep in metals within the framework of the nonlinear law (1).

We realize from Eq. (4) that the relaxation curves must be similar at different initial stress levels σ_0. This is confirmed by experiment in many instances.

In the case in point, Eqs. (3) and (4) acquire the following respective forms:

$$\varphi[\varepsilon(t)]=\sigma_0\left[1+\frac{\lambda t^{\alpha+1}}{\Gamma(\alpha+2)}\right] \tag{6}$$

$$\sigma(t)=\sigma_0\left[1+\lambda\int_0^t \vartheta_{\alpha}(-\lambda,\, t-\tau)\,d\tau\right] \qquad (\lambda>0) \tag{7}$$

The determination of the parameters α and λ and the function $\sigma = \varphi(\varepsilon)$ can be approached from two sides.

The set of creep curves can be used to construct a family of isochronous curves and to determine the parameters α and λ in Eq. (6) and to then construct the curve of the instantaneous stress state $\sigma = \varphi(\varepsilon)$ [3]. The theoretical relaxation curves are plotted on the basis of Eq. (7) with the values obtained for the constants α and λ.

On the other hand, the necessary characteristics can be determined from Eq. (7) by processing the relaxation curves. That is the way we chose in our work.

Zvonov et al. [4] considered a computer program for determining the characteristics of creep in linear hereditary-elastic materials. This computer method can be applied to the case of nonlinear behavior of a material for the purpose of determining relaxation characteristics.

The gist of the method is as follows. Instead of approximating the relaxation curve with the aid of Eq. (7), we proceed to take the Laplace-Carson transform of the relaxation curve, and the resulting transform is then approximated by the transform of Eq. (7), which appears in the form

$$\sigma(p) = \sigma_0\left[1 - \frac{\lambda}{p^{\alpha+1}+\lambda}\right], \qquad \sigma(p) = p\int_0^\infty \sigma(t)\, e^{-pt}dt \tag{8}$$

Here $p > 0$ is the transform parameter.

The quadratic method for optimizing the parameters (generalizing to the nonlinear case in the least-squares method) is then applied to the resulting transform, and the relaxation characteristics are determined without returning to the inverse transforms.

The method for determining the relaxation characteristics was programmed for a Minsk-2 digital computer.

Stress relaxation curves for copper [5], plotted for temperatures 20°, 165°, 235°C, were investigated (see Fig. 1a).

First, the hypothesis on the similarity of the relaxation curves was verified. The relaxation curves were plotted in $(\sigma_0 - \sigma_t)/\sigma_0$, t coordinates (Fig. 1b). This procedure yielded data which are in excellent agreement with the assumptions entertained. Each curve was then processed through the computer program. The results of the computer processing for those values are

	$T = 20$				$T = 165$		$T = 235$
$\sigma =$	15.58	8.54	5.93	3.18	9.50	7.50	5.85
$\alpha =$	−0.748	−0.812	−0.861	−0.765	−0.683	−0.627	−0.5015
$\lambda =$	0.421	0.386	0.379	0.405	0.147	0.158	0.1312

The averages of the relaxation parameters in Eq. (7) are

$$\alpha = -0.80, \quad \lambda = 0.398 \quad \text{at } 20°\,C$$
$$\alpha = -0.65, \quad \lambda = 0.157 \quad \text{at } 165°\,C$$
$$\alpha = -0.50, \quad \lambda = 0.131 \quad \text{at } 235°\,C$$

With those assumptions the parameters α and λ in Eq. (6), which describes the creep process in copper at those temperatures, will have the same values, and a family of isochronous curves at fixed t can be constructed when the curve of the instantaneous state $\sigma = \varphi(\varepsilon)$ is available, and these curves can then be used to plot the curves for creep in copper.

Since the values of constant strains ε_0 at which the relaxation experiments were staged are not indicated by Davis [5], the pattern of nonlinearity cannot be discerned nor the instantaneous extension curves constructed from the relaxation data. But inasmuch as the creep curves for the corresponding temperatures are available, the values of the parameters α and λ obtained for those temperatures can be used to construct the instantaneous loading curves $\sigma = \varphi(\varepsilon)$.

We illustrate this approach by the curve plotted for 235°C. We begin by plotting isochronous creep curves for copper at 235°C in σ, ε coordinates at different initial stress levels. These curves are shown in Fig. 1c in the form of a bundle of similar curves at fixed times indicated on the graph. The similarity of the isochronous creep curves confirms the validity of Eq. (1) for solving the problem posed.

Later on, an instantaneous loading curve was constructed on the basis of formula (6) by extrapolation with each isochronous creep curve at the parameter values obtained $\alpha = -0.50$ and $\lambda = 0.1312$. By specifying different values of σ, we obtain $\varphi(\varepsilon)$ values corresponding to the fixed time $t_1 = 20$ h. The same calculations are then carried out for another fixed $t_2 = 50$ h, etc. The $\varphi(\varepsilon)$ values so obtained and the averaged curve constructed on the basis of those values are presented in Fig. 1c. This curve is the one taken as the instantaneous loading curve $\sigma = \varphi(\varepsilon)$ reflecting the nonlinear behavior of the material. The slight spread of the values corresponding to the different isochronous curves in this instance serves as proof of the validity of the use of the values of the parameters α and λ found for describing creep.

Creep curves can be plotted for any σ = const from the resulting instantaneous loading curve. Substitution of the σ_0 values corresponding to the real curves in Eq. (6) and use of the $\sigma = \varphi(\varepsilon)$ graph aid in constructing the creep curves. These are plotted as dashed curves in Fig. 1d. The experimental curves are plotted as continuous curves. The correlation between the predicted data and experimental data is satisfactory. Subsequently, the $\sigma = \varphi(\varepsilon)$ curve so obtained can be approximated by any analytical dependence, e.g., a power-law dependence.

The results of the calculations indicate that the use of nonlinear hereditary equations with fractional-exponential kernels, applied to the description of stress relaxation in some metals and to the prediction of creep from relaxation data, has yielded positive results.

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LITERATURE CITED

1. Yu. N. Rabotnov, Creep in Structural Members [in Russian], Nauka, Moscow (1966).
2. Yu. N. Rabotnov, L. Kh. Papernik, and E. N. Zvonov, Tables of the Fractional-Exponential Function of Negative Parameters and Its Integral [in Russian], Nauka, Moscow (1969).
3. R. M. Goldhoff, "The application of Rabotnov's creep parameter," Proc. Amer. Soc. Testing Materials, 61 (1961).
4. E. N. Zvonov, N. I. Malinin, L. Kh. Papernik, and B. M. Tseitlin, "Determination of the creep characteristics of linear elastic-hereditary materials by digital-computer methods," MTT, No. 5 (1968).
5. E. A. Davis, "Creep and relaxation of oxygen-free copper," J. Appl. Mech., 10, No. 2 (1943).